and applying the relation between $[f_w]_{nc}$ and $[f_w]_{mx}$ as given by equation (10).

With a given $[f_w]_{mx}$ and C, $[f_w]_{nc}$ can be determined through equation (10). Once $[f_w]_{nc}$ is specified, $[-\theta'(0)]_{nc}$ can be solved from equations (8) and (9). Therefore, the free convection asymptote is obtained, from equation (22), for each corresponding $[f_w]_{mx}$.

To summarize, the influence of surface mass transfer on mixed convection over horizontal plates in saturated porous media has been studied analytically. Similarity solutions have been reported for the special cases for which the wall temperature, free stream velocity and injection or withdrawal velocity are a prescribed power function of distance. It is found that the heat transfer, in the form of free, mixed or forced convection, is enhanced by the withdrawal of fluid from the surface while it is decreased by the injection of fluid. Problems of this kind may be encountered frequently in the geophysical and geothermal applications. Solutions thus obtained, although applicable only to the injection and withdrawal of the same species, provide useful information when surface mass transfer due to chemical reactions has to be considered.

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Evaluation of spherical harmonics approximation for radiative transfer in cylindrical furnaces

NEVIN SELÇUK
Department of Chemical Engineering, Middle East Technical University, Ankara 06531, Turkey

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1. INTRODUCTION

IN A PREVIOUS paper [1], the accuracy of several radiation models for three-dimensional radiative heat transfer have been assessed by applying these models to the prediction of the distributions of radiative flux density and the radiative energy source term of a rectangular enclosure problem and by comparing these predictions with exact solutions [2].

A significant number of industrial furnaces and combustors are cylindrical in shape. Therefore, it is considered necessary to evaluate the radiation models produced earlier for cylindrical furnaces by applying them to the prediction of radiative flux density and source term distributions of a cylindrical enclosure problem based on data reported previously on a pilot-scale experimental furnace [3] and by comparing their predictions with exact values reported previously [4].

The radiation model to be tested is the spherical harmonics approximation derived for an axisymmetrical radiation field [5]. In this model the angular variation of intensity at a point is expressed by a series of spherical harmonics. By using the P_1 approximation (in which the series is truncated after the first four terms) and the equation of radiative transfer, the axisymmetrical radiation field within a grey, non-scattering medium is represented by three partial differential equations in the total incident flux density and the net radiant flux densities in the positive coordinate directions [6].

This model had previously been reduced to two-flux form and applied to the prediction of the behaviour of large-scale experimental furnaces and predicted temperature and radiative flux density distributions had been compared with experimentally determined data [6, 7]. However, it has been found impossible to decide whether discrepancies between predictions and measurements are attributable directly to the radiation model employed or to inaccuracies in the submodels used for the prediction of flow, reaction, etc.

The use of exact solutions for testing purposes provides a means for assessing the accuracy of predictions of a radiation model in isolation from the models of flow and reaction.

In this paper, therefore, the accuracy of the P_1 spherical harmonics approximation is tested by applying it to the prediction of the distributions of the radiative flux density

NOMENCLATURE

q	component of radiative flux density vector [W m^{-2}]
Q	source term for radiative energy [W m^{-3}]
R, Γ, Z	coordinate directions.

Superscript

$\tilde{}$	dimensionless.

and the source term of a cylindrical enclosure problem and by comparing its predictions with exact solutions reported previously [4].

2. THE TEST PROBLEM

The model under consideration has been tested by making predictions for a black-walled enclosure problem for which exact solutions had been produced earlier [4]. The enclosure problem is based on data reported by Wu and Fricker [3] on a pilot-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces.

The experimental furnace under consideration is a vertical cylinder fired from the bottom end wall with natural gas and operates under atmospheric pressure. The side walls are water cooled. A detailed description of the data obtained from the furnace and used as input data for the P_1 spherical harmonics approximation can be found elsewhere [4, 8].

3. NUMERICAL SOLUTION PROCEDURE

The partial differential equations representing the radiation model have been combined to give a second-order partial differential equation in terms of the total incident flux density. It has then been re-cast into finite-difference form by using the control volume approach. As the variation of gas and wall temperatures is axisymmetrical, the enclosure has been subdivided into 2×20 control volumes in the r- and z-directions, respectively. A medium grid point lies at the geometrical centre of each control volume and a surface grid point lies in the centre of each control volume face in contact with the walls of the enclosure. Hence the total number of medium and surface grid points are 2×20 and $(2 \times 2 + 20)$, respectively. The resulting sets of simultaneous algebraic equations have then been solved by the Gauss–Seidel iterative procedure.

4. EVALUATION OF THE MODEL PREDICTIONS

Point values of the dimensionless radiative energy source term and flux density produced by using the P_1 spherical harmonics approximation have been compared with the exact solutions reported previously [4, 8].

4.1. *Source term distributions*

Figure 1 illustrates the comparison between the exact values of the dimensionless source term and the distributions predicted by the model for grid points ($\tilde{R} = 0.25$, $\tilde{\Gamma} = 0$, $\tilde{Z}$) and ($\tilde{R} = 0.75$, $\tilde{\Gamma} = 0$, $\tilde{Z}$). These grid points represent the medium points at the centre of the row of control volumes nearest to the furnace axis and nearest to the side wall, respectively. It can be seen that good agreement is obtained and that the source term distributions nearer to the side wall show smaller variation along the length of the furnace. This is consistent with the uniform temperature distribution in the medium near the wall of the enclosure. The accuracy of the predicted source terms has been expressed in terms of maximum point and average absolute percentage errors which were found to be −11.33 and 9.04, respectively.

4.2. *Flux density distributions*

Figure 2 shows the comparison between the point values of the dimensionless flux density to the side wall predicted

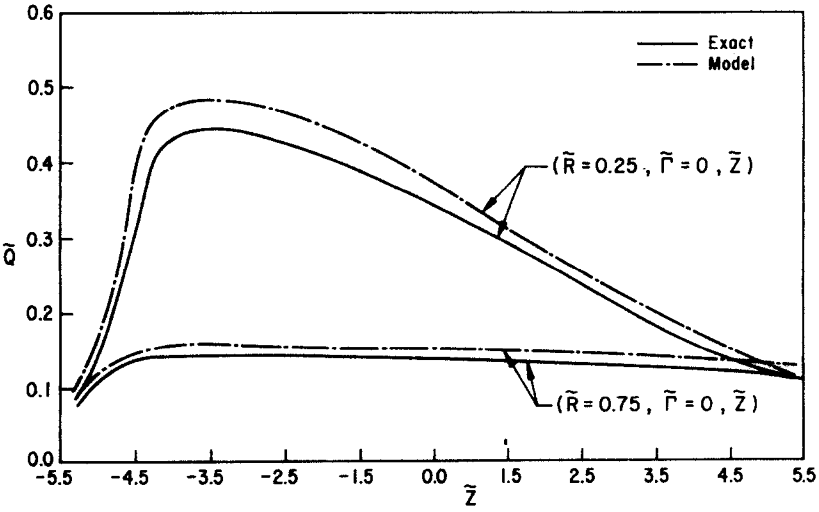

FIG. 1. Comparison between the exact values and model predictions of dimensionless radiative energy source terms.

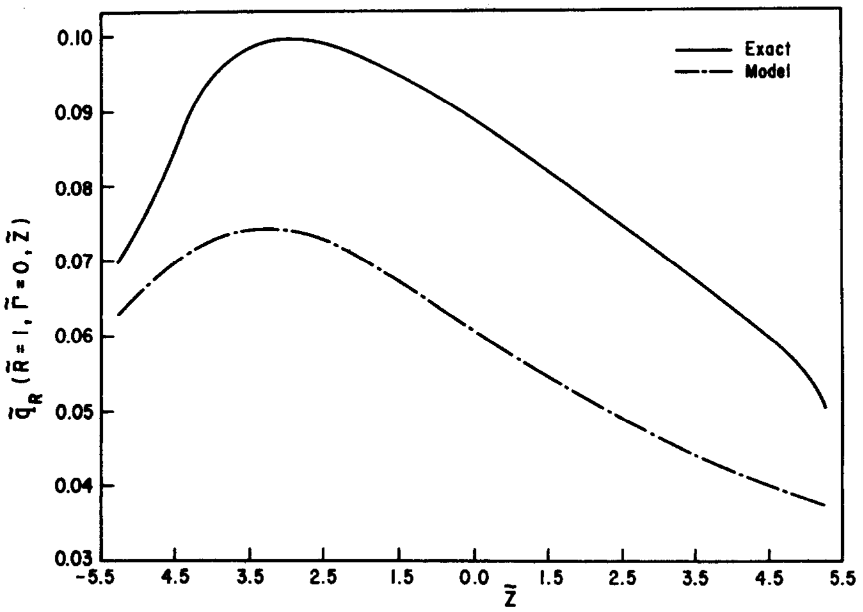

FIG. 2. Comparison between the exact values and model predictions of dimensionless flux densities to the side walls.

by the model and the exact solutions. It can be seen that the model underestimates the flux densities to the wall over the whole length of the enclosure. The maximum point and the average absolute percentage errors in the predicted flux densities are 28.10 and 24.15, respectively.

To provide a global check on the accuracy of the model predictions, the total rate of removal of radiative energy through the walls and the total rate of generation of radiative energy within the enclosed medium were calculated and compared with the exact values. Percentage errors in generated and removed radiative energy were found to be the same and equal to −10.06, implying that the model produces consistent results, although different from the exact values.

5. CONCLUSIONS

The P_1 spherical harmonics approximation for cylindrical enclosures filled with an absorbing–emitting medium of constant properties have been applied to the prediction of distributions of radiative flux density and energy source term of a black-walled enclosure problem. The problem is based on data reported previously on a pilot-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces. The accuracy of the model has been tested by comparing its predictions with exact solutions reported earlier in the literature. On the basis of comparisons it was concluded that the model produces accurate results with respect to the radiative energy source terms and underpredicts the radiative flux densities to the side wall.

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